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BASIC ALGEBRA RULES
**Exponent Simplification and Equations**

**Rules**

\[ a^m \times a^n = a^{m+n} \]

When multiplying like bases, we add the powers.

\[(a \times b)^m = a^m \times b^m \]

When raising parenthesis to a power, all elements are raised to that power.

**CAUTION!!** \((a + b)^m \neq a^m \times b^m\), \((a + b)^m = a^m + 2ab + b^m\)

\[ \frac{a^m}{a^n} = a^{m-n} \]

When we divide like bases with powers, subtract the bottom power from the top.

\[(a^m)^n = a^{mn} \]

When raising a power to another power, we multiply the powers.

\[\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \]

When raising a fraction to a power, both top and bottom are raised to that power.

\[\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n} \]

A fraction raised to a negative power is flipped.

\[a^{-n} = \frac{1}{a^n} \]

A number to a negative power is equal to one divided by that number and power.

\[a^0 = 1 \]

Any number to the power of 0 is equal to 1.

**Examples with Solutions**

\[2 \times x^{-3} = 2 \times \left(\frac{1}{x^3}\right) = \frac{2}{x^3}\]

\[2y^3 \times 4y^2 = (2 \times 4) \times y^{(3+2)} = 8y^5\]

\[H^4 \times y^{12} \times H^3 \times y^5 = H^{(4+3)} \times y^{(12+5)} = H^7 y^{17}\]

\[(3x^2 \times y^4)^2 = 3^2 \times x^{2 \times 2} \times y^{4 \times 2} = 9x^4 \times y^8\]

\[\frac{3x^5y^2s^2}{6xys^2} = \frac{1}{2}x^{5-1} \times y^{1-2} \times s^{2-5} = \frac{x^4}{2ys^3}\]
Practice Problems:

Simplify the following expressions:

1. \( x^6 \cdot x^3 \)
2. \( y^8 \div y^5 \)
3. \( (a^3 b)^7 \)
4. \( (p^3 \cdot q^9)^{-2} \)
5. \( (a^8)^{1/2} \)
6. \( (3ab^4)^3 \)
7. \( \left( a^4 \right)^0, a \neq 0 \)
8. \( \frac{-3x^7 y^{-8} z^{9} z^{1}}{9x^{-3} y^{-5} z^{2}} \)
9. \( \frac{b^{-2}}{\frac{b^{-5}}{b^{-3}}} \)
10. \( \frac{28x^3 y^3 z}{7x^{-4} z^2 y^4} \)
**Square Root Manipulation**

Rules

\[ \sqrt{a} = \sqrt[2]{a} \]  
A square root with no specified number is assumed to be 2 (square root).

\[ \sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{a \times b} \]  
When multiplying with roots, numbers can be split into their own roots.

**CAUTION!!** \[ \sqrt{a} + \sqrt{b} \neq \sqrt{a + b} \]

\[ \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \]  
When dividing with roots, numbers can be split into their own roots.

\[ a^{\frac{m}{n}} = \sqrt[n]{a^m} \]  
Roots can be written as fractional powers.

\[ a^{-n} = \frac{1}{a^n} \]  
A number to a negative power is equal to one divided by that number and power.

Definition

Rationalizing the denominator means that there are no roots left in the denominator. This can be done but multiplying by the conjugate of the root on top and bottom for two term problems or just the root itself for one term problems.

Example

Simplify \( \frac{6}{\sqrt{12}} \)

Solution

\[
\frac{6}{\sqrt{12}} = \frac{6\sqrt{12}}{12} = \frac{\sqrt{12}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}
\]

Example

Simplify \( \frac{1}{2+\sqrt{7}} \)

Solution

\[
\frac{1}{2+\sqrt{7}} \cdot \frac{2-\sqrt{7}}{2-\sqrt{7}} = \frac{2-\sqrt{7}}{4-7} = \frac{2-\sqrt{7}}{-3}
\]
Practice Problems:

Simplify the following expressions:

1. \( \frac{\sqrt{32}}{(\sqrt{8})^2} \)

2. \( \sqrt{\frac{4}{3} + \frac{(\sqrt{32})^2}{3}} \)

3. \( \sqrt[3]{81x^2y^2z} + \sqrt[3]{z} \)

4. \( -\frac{53x^2}{-x} + \sqrt{(-x)^2} + -50\sqrt{x^2} \)

Rationalize the Denominator:

5. \( \frac{5-\sqrt{7}}{\sqrt{7}} \)

6. \( \frac{4+\sqrt{2}}{\sqrt{8}} \)

7. \( \frac{4}{3+\sqrt{5}} \)

8. \( \frac{1+\sqrt{6}}{2-\sqrt{3}} \)


**Imaginary Numbers**

**Rules**

Imaginary numbers are sometimes referred to as complex numbers and are present when you have even roots of a negative number. For example: \( \sqrt{-9} \).

Little \( i \) is what is used to represent an imaginary number. Specifically, \( i = \sqrt{-1} \). So, the above number, \( \sqrt{-9} \) would instead be written as \( 3 \ast i \).

Imaginary numbers are usually written in standard form which is written below and is

\[
\text{standard form} \quad a + b \, i
\]

where \( a \) and \( b \) are real numbers. Imaginary numbers written in standard form make it easy to find the conjugate of any imaginary number.

**Definition**

The **conjugate** of an imaginary number is when you repeat all numbers present, but switch the sign of any term that has an imaginary present. The conjugate is used in the same way as rationalizing the denominator of a fraction with a root in the bottom.

**Example**

Find the conjugate of \( 3 - 7 \, i + 2 \sqrt{-2} \).

**Solution**

\[
3 - 7 \, i + 2 \sqrt{-2} \rightarrow 3 + 7 \, i - 2 \sqrt{2} \, i = 3 + (7 - 2 \sqrt{2}) \, i
\]

**Example**

Simplify \( (3 \, i^2 + 2)(i + 1) \).

**Solution**

\[
(3 \, (-1) + 2)(i + 1) = (-3 + 2)(i + 1) = -i - 1
\]
Practice Problems:

Simplify the following problems (ie. Write in a + b*i form); Use the conjugate where necessary

1. \( \frac{2}{3+2i} \)
2. \( \frac{4}{3-7i} \)
3. \((3 + 2i)(-3 - 2i)\)
4. \( \frac{5i+2-6i}{i+1} \)
5. \( \frac{-2+4i}{3-i^3} \)

Find each of the following and simplify

a. \( x + y \)  
b. \( x - y \)  
c. \( x \times y \)  
d. \( \frac{x}{y} \)

6. \( x = 3i, y = 2 - i \)
7. \( x = \frac{1}{2} - i\sqrt{3}, y = \frac{1}{5} + 3i\sqrt{3} \)

Solve for each x value******

8. \( 0 = 2x^2 - x + 8 \)
9. \( 0 = -3x^2 - 18 \)
10. \( 0 = -5t^2 + 2t - 3 \)

******Only do so if familiar with the quadratic formula.

If not, reference section QUADRATICS AND HIGHER LEVEL ALGEBRA on page 42
ALGEBRAIC EQUATIONS
**Absolute Value Equations**

**Rules**

Absolute value of a number is described as a “distance from zero” or the “positive” or the “magnitude” of a number. Any negative numbers absolute value is changed to a positive.

This means that each equation with an absolute value has a possibility of more than one solution, both the positive and the negative.

**Example**

\[ |7| = |-7| = 7 \]

As you can see, the absolute value of 7 and \(-7\) are both equal to 7.

We can look at it graphically as well below

Let’s look at this example only with a couple of changes.

\[ |x + 4| = 7 \]

Just as in the example above, we should have two ways to get the answer 7:

\[ x + 4 = 7 \quad AND \quad -(x + 4) = 7 \]

If we solve each equation, we see that \( x = 3 \) or \( x = -11 \).

**Solution**

Therefore, the solution is \( x = 3 \) or \( x = -11 \)
**Practice Problems:**

Simplify the following:

1. $|-5|$

2. $-|5|)$

Solve the following equations for $x$:

3. $|2x + 4| = 12$

4. $x < |-2|$

5. $5 - |3x| = 5$

6. $|4x| = 12$

7. $|2x - 2| = 4$

8. $|2x^2| = 8$

9. $\left|\frac{x}{3}\right| = 5$

10. $4|x| = 20$
Logarithms

Rules

\[ \log(a) + \log(b) = \log(ab) \]

\[ \log(a) - \log(b) = \log\left(\frac{a}{b}\right) \]

\[ \log(x^a) = a \log(x) \]

\[ \log_b a = x \] can be equivalently written as \[ b^x = a \]

\[ \ln(e) = 1 \]

\[ e^{\ln(x)} = x \]

Example

Rewrite the equation as a log: \[ 5^x = 52 \]

Solution

\[ \log_5 52 = x \]

Example

\[ e^x = 7 \]

Solution

\[ \ln(e^x) = 7 \quad \quad x \cdot \ln(e) = 7 \quad \quad x = 7 \]
Practice Problems:

Simplify the following expressions

1. \( \log_2 8 \)
2. \( \ln(e) \)
3. \( \log_2 -10 \)
4. \( \log_{10} 1 \)
5. \( \ln \left( \frac{10}{10} \right) \)
6. \( \log_{10}(10 \times 10) \)

Use Log Rules to rewrite the following expressions

7. \( \log(15 \times 3) \)
8. \( \log \left( \frac{15}{3} \right) \)
9. \( \log(10^{10}) \)

Solve the following for \( x \)

10. \( \left( \frac{a^3}{a^{-1}} \right)^9 = \frac{a^{4x}}{a^{10}} \)
11. \( 7^{11x-5} = \frac{49x}{7^4} \)
12. \( \frac{\ln(7)}{-4} = -6 \)
GRAPHING
**Higher Level Graphing**

Graphing Polynomials

Polynomials always have one less turn in them than the highest exponent. The graph to the right is called a parabola and has one turning point (or one maximum or minimum).

This is because the highest exponent is a two. For a polynomial that is something like $x^5 + x$, there would $5 - 1 = 4$ total turning points (max/min) possible.

This graph to the left is a cubic which means it can have up to 2 turning points. Notice the difference between the graph to the left, and the graph below. The graph to the left is centered at the origin and the intercept can be found by setting $y = 0$ and solving for $x$. In this case, it yields an intercept of $x = 0$.

Notice this graph’s equation is a bit more complex. However, we could graph by looking at the intercepts again. Setting $y = 0$, we obtain x-intercepts of $x = 0$ and $-3$. In order to really be able to make a sketch of the graph, we can then tell whether the graph is positive or negative on either side of the intercepts by choosing x values and seeing what the y values are. For example, if we choose $x = -1$, we can see $y = \frac{1}{3}(-1) + (1) = \frac{2}{3}$. Since y is positive, we know the maximum is in between the x values of $-3 < x < 0$. Similar tests can determine the other sides.
Vertical and Horizontal Shifts

Any graph can be shifted vertically or horizontally. There is a subtle difference between how to tell which shift is going to occur. In general, if the number is by itself (not attached to the x) it will be a vertical shift. Positive numbers correspond to a shift up, and negative numbers correspond to a shift down. The graph to the right shows a vertical shift with the formula for a given parabola. The dotted line is the original, and the solid is the shifted.

A horizontal shift happens when a number is added or subtracted to the x-value on the inside of the parentheses. For example, a parabola can have the form

\[ y = (x - h)^2 + k \]

However, in this case, the shift goes the opposite direction that you would think because the form is \((x - h)\). This means that negative numbers shift the graph to the right, and positive numbers shift the graph to the left. The examples just to the left show both types of horizontal shifts. Notice the different signs in the formulas and how they move the graph.

It is very important to remember that the horizontal shifts move opposite the sign while the vertical moves with the sign.
Reflections

Reflections about the x axis can also occur. This happens if the sign in front of the x (or in front of whatever term contains the x) is negative. The graph below shows the formula as well as the result of a reflection about the x axis.

Of course, you can have combinations of all horizontal shifts, vertical shifts and reflections in one graph. An example is shown below of this.

Example
**Practice Problems:**

Graph each of the following

1. \( y = \sqrt{x} + 1 \)

2. \( y = 4x - 1 \)

3. \( (x - 5)^2 = -(y + 1) \)

4. \( y = (x - 2)^2 + 2 \)

5. \( x = 0 \)

6. \( y = -\frac{3}{2} \)

7. \( 20x^2 + 20y = 10 \)

8. \( y = (x + 1)^3 + 4 \)

9. \( y = -(x + 2)^2 + 1 \)

10. \( y = 4 - |x - 2| \)
Graphing Inequalities

Example

Graph the following inequality.

\[ y \geq -3x + 6 \]

Solution

First, graph the equation. Then, shade the region above or below the line:

- If “\( y \) is greater than,” shade above since above \( y \) values are greater than those on the line
- If “\( y \) is less than” shade below since all the \( y \) values below will be less than on the line

Check a test point to make sure it works (for example the point \( x = 0 \) and \( y = 0 \))

\[ 6(0) + 2(0) \geq 12 \]

\( 0 \not\geq 12 \) Decision: The point (0,0) should not fall in the shaded region

Notice:

Since the equation given at the top of the page states that “\( y \) is greater than or EQUAL TO -3 \( x \) + 6” the region above the graph is shaded.

The line is SOLID since it could also be equal to. For any inequality that is strictly greater than or less than and NOT EQUAL TO, the line should be DOTTED.
**Practice Problems:**

Solve each inequality and graph them

1. \( y \leq x \)
2. \( y < 4x - 1 \)
3. \( 3y - 12 \leq -6x \)
4. \( y > \frac{1}{2}x + 3 \)
5. \( x \leq 0 \)
6. \( y \geq -2.5 \)
7. \( x - 6y \leq 7 \)
8. \( y > x^3 \)
9. \( y \leq -(x + 2)^2 + 1 \)
10. \( y < |x - 2| - 3 \)
**Range and Domain**

Definitions

The **domain** of a function is all x-values that have a corresponding y-value for a given function.

The **range** of a function is all of the possible y values of a given function.

Example

Given the following graph, determine the domain and range; state whether or not it is a function.

\[ y = (x - 2)^2 - 1 \]

Solution

The given shape is called a parabola and extends indefinitely to the left and right as indicated by the arrows. This suggests that if we choose any \( x \)-value, then we will be able to find a corresponding y value on the graph. Therefore, the **domain** consists of all real numbers. The graph also shows that -1 is the minimum y-value, and any y-value greater than is in the range. Any y value below is not in the range since those y values can never be obtained by that function. Hence, the **range** consists of all y-values greater than or equal to -1, or in interval notation, \([-1, \infty)\). The solution is shown visually here.

We also know that this is a function because it passes the vertical line test. Any graph that does not pass the vertical line test is not a function.
Practice Problems:

Determine the domain and the range of the following functions

1. \( f(x) = 2^x \)

2. \( f(x) = \frac{2x^2}{x^2 - 9} \)

3. \( f(x) = \sqrt{x} + 5 \)

4. \( f(x) = \frac{4}{\sqrt{x}+5} \)

5. \( f(x) = \sqrt{x-2} + 3 \)

6. In the set of real numbers, what is the domain of \( f(x) = \frac{4x}{\sqrt{x-4}} \)?

Without a calculator, graph each of the following functions and specify the domain and range

7. \( y = e^x \)

8. \( y = e^{-x} \)

9. \( y = \frac{2x^3 - 7}{3x^3 + 24} \)

10. \( y = \frac{x^3 + 3}{x^3 - 5} \)

11. \( y = \cos(x) \) ***

12. \( y = \sin(x) \) ***

13. \( y = \tan(x) \) ***

***For these problems, if you need a trigonometry review first, come back to these once you are finished with the TRIGONOMETRIC GRAPHING section on page 68.
FUNCTIONS
**Even and Odd Functions**

Definitions

An *even function* is one that follows that

\[ f(-x) = f(x). \]

Properties of even functions are that they are symmetric about the x axis. When looking at a graph, imagine folding the paper along the y axis and if it lies on top of itself, the function is even. The example to the right is that of an even function.

An *odd function* is one that follows that

\[ f(-x) = -f(x). \]

Properties of odd functions are that they are symmetric about the origin. When looking at a graph, imagine folding the paper along the y axis and then flipping it over the x-axis to get the image on the other side. The example to the left is that of an even function. The dotted line is the image of the flip along the y axis which is then turned upside down to get the image on the other side.

You can algebraically determine if a function is even, odd or neither by replacing x with \(-x\) and looking at the result.

**Example**

\[ f(x) = |x| \]

**Solution**

\[ f(-x) = |-x| = |x| = f(x), \] therefore \( f(x) \) is an even function.
Practice Problems:

Determine whether the function is even, odd or neither by looking at the graph.

1. 

2. 

3. 

4. 
Determine whether the following functions are even, odd or neither algebraically

5. \( f(x) = -|x| + 7x^2 \)

6. \( f(x) = -(x)^3 \)

7. \( f(x) = x^3 + 3x \)

8. \( f(x) = (x - 4)^3 \)

9. \( f(x) = (x^2 + 2)(x^2 - 4) \)

10. \( f(x) = -x^3 + 1 \)
**Inverses of Functions**

**Definition**

An inverse function can be found by switching $x$ and $y$ and then solving for $y$. Essentially, a function and its inverse undo each other, just like multiplication undoes division. If you apply a function $g$ and then a function $f$ and they are inverses of each other, you get what you started with.

An inverse of a graph is usually written as $f^{-1}(x)$. Graphically, it can be seen as a reflection about the line $y = x$. An example of that is just below.

**Example**

Find the inverse of $y = -\frac{1}{3}x + 1$

**Solution**

Algebraically:

By switching $x$ and $y$ you obtain: $x = -\frac{1}{3}y + 1$, then solving for $y$ you get the answer that $f^{-1}(x) = -3x + 3$

Graphically:

Imagine folding along the gray line; the inverse of the red dotted line is the blue line as shown.
**Practice Problems:**

Answer the following with the information given in each.

1. Suppose that \( f(x) \) and \( g(x) \) are a pair of inverse functions.
   a. If 13 is in domain of \( g(x) \), find \( f(g(13)) \)
   b. If \( \sqrt{5} \) is in the domain of \( f \), find \( g(f(\sqrt{5})) \)

2. Assume that the domain of \( f \) and \( f^{-1} \) is \((-\infty, \infty)\). Solve the equation for \( x \).
   a. \( f^{-1}(2x + 3) = 5 \); \( f(5) = 13 \)
   b. \( 7 + f^{-1}(x - 1) = 9 \); \( f(2) = 6 \)

3. Solve the following equation for \( x \), given that the domain of both \( f \) and \( f^{-1} \) is \((-\infty, \infty)\) and that \( f(1) = -2 \)
   \[ 3 + f^{-1}(x - 1) = 4 \]

4. Let \( f(x) = \frac{x - 1}{3x + 5} \). Find \( f^{-1} \).

5. The graph of a function \( f \) consists of the line segment joining the points \((-2, -3)\) and \((-1, 4)\). Sketch a graph of \( f^{-1} \).

6. Verify that the given pairs of functions are inverse functions:
   a. \( f(x) = -3x \); \( g(x) = \frac{x}{3} \)
   b. \( f(x) = 4x - 1 \); \( g(x) = \frac{1}{4}x + \frac{1}{4} \)
   c. \( g(x) = \sqrt{x} \); \( h(x) = x^2 \)
   d. \( f(x) = \frac{1}{3}x + 2 \); \( g(x) = 3x - 6 \)
Composite Functions

Examples

Functions can be multiplied, added, subtracted and divided, just like numbers can. The process you follow to solve them is the same as you would when simplifying an expression. For example,

\[ f(x) = 3x + 2 \quad \text{and} \quad g(x) = x + 2 \]

Then \( (f \cdot g)(x) = f(x) \cdot g(x) = (3x + 2) \cdot (x + 2) = 3x^2 + 8x + 4 \)

It is also possible to have one function nested into another. When you have something such as

\[ f(x) = 3x + 2 \]

where \( x = 3 \)

Then \( f(3) = 3 \cdot 3 + 2 = 11 \).

The same thing can happen with a function instead of just the number three. The process is exactly the same. Let’s instead say

\[ h(x) = 2x^2 + 3x \]

Then, let’s find the function for \( f(h(x)) \). Just as we would plug in 3 for \( f(3) \) we will plug in \( 2x^2 + 3x \) every place there is an \( x \) in \( f(x) \) and then simplify.

\[ f(h(x)) = 3(2x^2 + 3x) + 2 = 6x^2 + 9x + 2 \]

A little bit of notation is important. The above expression, \( (h(x)) \), can also be written as \( (f \circ h)(x) \). These two things are asking the same thing and is read aloud as "\( f \) of \( h \) of \( x \)."

CAUTION!!!!! Make sure you use parentheses when plugging in one function into another

\[ f(x) = -x + 2 \quad \text{and} \quad g(x) = 2x - 7 \]

Correct Answer: \[ f(g(x)) = -(2x - 7) + 2 = -2x + 9 \]

Common Mistake: \[ f(g(x)) = -2x - 7 + 2 = -2x - 5 \]
**Practice Problems:**

Compute each expression, given that the function \( f, g, h, k, \) and \( m \) are defined below

\[
f(x) = 2x - 1 \quad \quad \quad \quad \quad k(x) = 2 \quad \text{for all } x
\]
\[
g(x) = x^2 - 3x - 6 \quad \quad m(x) = x^2 - 9 \quad \quad h(x) = x^3
\]

1. \((f + g)(x)\)
2. \((f - g)(x)\)
3. \((k * h - g)(x)\)
4. \((f * g)(x)\)
5. \((f - m)(x)\)
6. \((f * h)(x)\)
7. \(\left(\frac{L}{h}\right)(x)\)

. Compute the following given that

\[
f(x) = \frac{3x-4}{3x+3} \quad \text{and} \quad g(x) = \frac{x+1}{x-1}
\]

8. \((f \circ g)(x)\)
9. \(f(g(t))\)
10. \(g((f)(y))\)
11. \((g \circ f)(x)\)

Express each of the following functions as a composition of two of the given functions.

12. \(\sqrt[3]{2x+1}\)
13. \(2 \sqrt[3]{x} + 1\)
14. \(\frac{1}{2x+1}\)
15. \(|3x - 1|\)
QUADRATICS
Simplification of Fractions using Quadratic Factoring

Rules

Essentially the goal is to just use factoring of quadratics to simplify these algebraic expressions. Recall that to expand something is looks like this:

**Foil Method:** (First Outside Inside Last)

\[(x + 2)(x + 4) = x^2 + 4x + 2x + 8\]

Then combine like terms to get \[x^2 + 6x + 8\].

Remember our goal is to go backward. So starting with the equation just above, notice that the last term is 8 and the Last term in each set of parentheses has to multiply to 8. The options we’d have are 8 and 1 OR 2 and 4. The way we decide which is the correct set of numbers is that we choose which combination will ADD to the middle number. In this case, \(4 + 2 = 6\) therefore we obtain \((x + 4)(x + 6)\).

Example

\[
\frac{x^2+4x+4}{x^2−4}
\]

Solution

\[
\frac{x^2+4x+4}{x^2−4} = \frac{(x+2)(x+2)}{(x+2)(x−2)} = \frac{x+2}{x−2}
\]
Practice Problems:

Simplify the following:

1. \( \frac{x-6}{x+5} \cdot \frac{x^2-25}{x^2-11x+30} \)

2. \( \frac{x+7}{x+8} + \frac{-x^2-8x-7}{x^2+9x+8} \)

3. \( \left( \frac{x^2-36}{x-6} \right) \left( \frac{x-5}{x^2-25} \right) \)

4. \( \left( \frac{x^2+12x+32}{x^2+15x+56} \right) \left( \frac{x+7}{x^2+10x+24} \right) \)

5. \( \frac{x^2+x-6}{x^2-7x+10} \div \frac{x^2-17x+60}{x^2+2x-3} \)

6. \( \frac{x^2+7x-30}{x^2-5x-50} \div \frac{x-3}{x^2-5x-50} \)

7. \( \frac{x^2+3x-18}{x^2+15x+54} \div \left( \frac{x^2-6x+9}{x^2+6x-27} \right) \)

8. \( \frac{2x^2-3x-9}{x^2-9} \cdot \left( \frac{x+3}{6x^2+11x+3} \right) \)

9. \( \frac{x^2-4}{2x^2+5x+2} \cdot \left( \frac{7x^2-19x-6}{x^2-5x+6} \right) \)

10. \( \frac{x^2-5x+6}{x^2-11x+18} + \left( \frac{5x^2+22x+8}{x^2-5x-36} \right) \)
Quadratics and Higher Level Algebra

Formulas

For any given equation of the form \( ax^2 + bx + c = 0 \), the quadratic formula can be used to solve for the variable. The solution is given by

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

For factoring cubics, there are two different formulas to memorize. They are called the difference of cubes formulas and are given by:

\[
a^3 - b^3 = (a - b)(a^2 + ab + b^2)
\]

- same sign
- opposite sign

\[
a^3 + b^3 = (a + b)(a^2 - ab + b^2)
\]

- same sign
- opposite sign

Example

\[2x^3 + 128y^3\]

Solution

\[
2x^3 + 128y^3 = 2(x^3 + 64y^3)
\]

\[
= 2[(x)^3 + (4y)^3]
\]

\[
= 2[x + 4y][x^2 - (x)(4y) + (4y)^2]
\]

\[
= 2(x + 4y)(x^2 - 4xy + 16y^2)
\]
Practice Problems:

Answer the following

1. Given $x^2 + 4x + 4$, what is the a, b, and c that you would use for the quadratic formula?

Solve for x

2. $x^2 + 4x + 4 = 0$
3. $x^4 - 4x^2 + 4 = 0$
4. $x^4 + 4x^2 + 3 = 0$
5. $x^3 + 3x^2 = 0$
6. $x^3 - 27 = 0$
7. $x^2 - 5x + 4 = 0$

Answer the following

8. Factor $x^3 - 27$
9. Expand $(x + 3)(x^2 + 3x + 9)$
10. Given $x^2 + 8x + y$, what must be the y be in order for the equation to be factored?
POLYNOMIAL LONG DIVISION
Polynomial Long Division

Example

Polynomial long division is used for simplifying expressions that have a higher power of \( x \) on the top of a fraction than the bottom.

Divide \( \frac{2x^3 - 5x^2 + x - 10}{x^2 - 4x + 1} \)

For this, we will follow essentially the same process as long division for numbers:

\[
\begin{array}{c|cc}
  \multicolumn{1}{l}{2x} & 2x^3 & -5x^2 + x - 10 \\
  \hline
  x^2 - 4x + 1 & 2x^3 & -8x^2 + 2x \\
  \hline
  & -2x^3 + 8x^2 - 2x & \\
  & \hline
  & 3x^2 - x - 10 & \\
  & \hline
  & 3x^2 - 3x + 3 & \\
  & \hline
  & 4x - 13 & \\
\end{array}
\]

Solution

Once you can no longer keep going, the process is complete. The piece left at the bottom is called the remainder. To write the full solution the expression above the division bar is written followed by the remainder that is written still in a fraction like this:

\[
\frac{2x^3 - 5x^2 + x - 10}{x^2 - 4x + 1} = 2x + 3 + \frac{11x - 13}{x^2 - 4x + 1}
\]
Practice Problems:

Use polynomial long division to simplify the following

1. $x^2 - 1$ divided by $x - 1$

2. $\frac{x^3+4x^2+3x}{x^2+x}$

3. $\frac{5x^3-13x^2-5x-3}{x-3}$

4. $\frac{4x^{20}-10x^7+5x^3}{x^3}$

5. $\frac{x^3-8x+2}{x-3}$

6. $\frac{9x^8+x^4+3}{x^3}$

7. $\frac{5x^5+x^3+2x+5}{x^4+1}$
GEOMETRY
Geometry

Formulas

Squares
- **Perimeter**: \( P = 4 \times x \), where \( x \) is the length of each side of the square since all sides are the same length.
- **Area**: \( A = x^2 \), since the area is the length times width which for a square is \( x \times x = x^2 \).

Rectangles
- **Perimeter**: \( P = 2 \times L + 2 \times W \) where \( L \) is the length and \( W \) is the width.
- **Area**: \( A = L \times W \)

Triangles
- **Perimeter**: \( P = A + B + C \), where \( A \), \( B \) and \( C \) are the lengths of the sides of the triangle.
- **Area**: \( A = \frac{1}{2} \times Base \times Height \)

Circles
- **Diameter**: \( D = 2 \times R \), where \( R \) is the radius of the circle
- **Circumference**: \( C = 2 \times \pi \times R \)
- **Area**: \( A = \pi R^2 \)
Practice Problems:

Find the area of the given shaded region:

1.

![Rectangle with dimensions](image1)

2.

![Square with dimensions](image2)

3.

![Triangle with dimensions](image3)
4. 

![Diagram of a rectangle with dimensions labeled as 2H, 2y, y, y, and y.] 

5. 

![Diagram of a rectangle with dimensions labeled as 2x, 7x, and a circle inside.] 

In the following problems, use the given information to solve the problem.

6. 

If the length of this rectangle is 4 times the width, and the area is 36 units squared, what is the width?

![Diagram of a rectangle with dimensions labeled as x and x.] 

7. 

If the hypotenuse of the triangle below is $\sqrt{2}$, what is the area of the triangle?

![Diagram of a right triangle with sides labeled as x and x.]
8. The area of the figure below is 144 units squared. What is the value of x?

![Diagram of a figure with sides 3x, 5x, 2x, and 3x]

9. If \( z = 2 \), find the Area and the Perimeter.

![Diagram of an L-shaped figure with sides z, z, z, and 5z]

10. Given that the circumference of the circle is \( 8 \pi \), what is the area of the shaded region?

![Diagram of a circle with square borders]
TRIGONOMETRY AND TRIGONOMETRIC EQUATIONS
Unit Circle and Basic Trigonometry

Unit Circle

For Radians:

Any where you see blue, think of breaking the circle into integer fractions of $\frac{1}{6}$.

Any where you see red lines, think of breaking the circle into integer fractions of $\frac{1}{4}$.

For Degrees:

Any where you see red, it is a multiple of $45^\circ$.

Anywhere you see blue, it is a multiple of $30^\circ$.
Useful Formulas

**Reciprocal Identities:**

\[
\frac{1}{\sin(x)} = \csc(x) \\
\frac{1}{\cos(x)} = \sec(x) \\
\frac{1}{\tan(x)} = \cot(x) \\
\frac{\sin(x)}{\cos(x)} = \tan(x)
\]

**Pythagorean Identities:**

\[
\sin^2(x) + \cos^2(x) = 1 \\
\sin^2(x) + \cos^2(x) = 1 \\
1 + \cot^2(x) = \csc^2(x) \\
\sin^2(x) + \cos^2(x) = 1 \\
\tan^2(x) + 1 = \sec^2(x)
\]

****Divide first identity
to get the other identities

To convert from radians to degrees use the equivalence that \(180^\circ = \pi \text{ radians}\) and use units to cancel the one you want to get rid of.
Practice Problems:

Solve each of the following

1. \( Sin \left( \frac{5\pi}{3} \right) \)

2. \( Cos(135^\circ) \)

3. \( Csc \left( \frac{5\pi}{3} \right) \)

4. \( Tan(2\pi) \)

5. \( Cos(x) \) is positive in which of the four quadrants?

6. Convert \( \frac{7\pi}{15} \) from radians into degrees.

7. Convert \( 408^\circ \) into radians.

8. Convert \( \frac{4\pi}{3} \) from radians into degrees.

9. Simplify \( \frac{Sin(x) \cdot Sec(x)}{Tan(x)} \) using the reciprocal identities.

10. Fill in the unit circle on your own paper without looking.
Solving Trigonometric Equations

Use what you learned about the Unit Circle to help you solve these problems.

Example

Solve: $\tan x \sin^2 x = 2 \tan x \quad x \in [0, 2\pi]$

$$\tan x \sin^2 x = 2 \tan x$$
$$\tan x \sin^2 x - 2 \tan x = 0$$
$$\tan x (\sin^2 x - 2) = 0$$

At this point we know that:

$$\tan x = 0 \quad \text{or} \quad \sin^2 x - 2 = 0$$

$$\sin^2 x = 2$$
$$\sin x = \pm \sqrt{2}$$

Now, $\tan x = 0$ implies that $x = 0, \pi, 2\pi$ (see graph). Since the sine function has is bounded by the values of $+1$ and $-1$ (i.e. the range of $\sin(x)$ is from $-1$ to $1$), the values for $\sin x = \pm \sqrt{2}$ has no solutions.

Solution

Thus the answer $x = 0, \pi, 2\pi$ is the only solution.
Practice Problems:

Solve the following equations

1. \( \frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = 4 \) \hspace{1cm} 0 \leq x \leq 2\pi

2. \( 2 \sin x - 1 = 0 \) \hspace{1cm} 0 \leq x \leq 2\pi

3. \( \cos(x) = \frac{1}{2} \) \hspace{1cm} 0 \leq x \leq 2\pi

4. \( \cos \left( 5x + \frac{\pi}{2} \right) = \frac{\sqrt{2}}{2} \) \hspace{1cm} 0 \leq x \leq 2\pi

5. \( 2 (\sin x)^2 + 3 \cos x - 3 = 0 \) \hspace{1cm} 0 \leq x \leq 2\pi

6. \( 2 (\sin x)^2 + \sin x - 1 = 0 \) \hspace{1cm} 0 \leq x \leq 2\pi

7. \( 3 (\cos t)^2 - 5 \sin t - 4 = 0 \) \hspace{1cm} 0 \leq t \leq 360

8. \( 4(\sin x)^2 - 1 = 0 \) \hspace{1cm} 0 \leq x \leq 2\pi

9. \( 2 \cos(3x - 1) = 0 \) \hspace{1cm} 0 \leq x \leq 2\pi

10. \( \sin(2x + \frac{\pi}{2}) = \frac{\sqrt{3}}{2} \) \hspace{1cm} 0 \leq x \leq 2\pi
GEOMETRIC TRIGONOMETRY
Geometric Trigonometry

Right Triangles

**Pythagorean Theorem** \( A^2 + B^2 = C^2 \)

**SOH** \( \sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{A}{C} \)

**CAH** \( \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{B}{C} \)

**TOA** \( \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{A}{B} \)

Not right triangles

Given the triangle to the right, the following applies:

**Law of Sines** \( \frac{\sin(A)}{A} = \frac{\sin(B)}{B} = \frac{\sin(C)}{C} \)

**Law of Cosines** \( a^2 = b^2 + c^2 - 2 \ b \ c \ \cos(A) \)

\( b^2 = a^2 + c^2 - 2 \ a \ c \ \cos(B) \)

\( c^2 = a^2 + b^2 - 2 \ a \ b \ \cos(C) \)
Practice Problems:

Find Sin θ, Sec θ, Tan θ for the following

1.

![Diagram 1](image1)

2.

![Diagram 2](image2)

Find θ for the following

3.

![Diagram 3](image3)
4. Find the value of $x$ in the following figure.

5. Find the value of $x$ in the following figure.

6. Find the value of $x$ in the following figure.

7. Find the value of $x$ in the following figure.
8.

\[6\sqrt{2}

45^

X

9.

\[3

60^

4

X

10.

\[\sqrt{3}

30^

\theta

X

1
TRIGONOMETRIC GRAPHING
Trigonometric Graphing

Graphs to Know

*** Compare Secx and Csecx
Graph with Shifts

Just as in the previous Higher Level Graphing section, trigonometric graphs are subject to shifts as well. There are FOUR different things to consider when graphing trigonometric functions.

The general form of a trigonometric equation (using cosine as a specific function) is given by

$$ f(x) = A \cdot \cos(b \cdot x - h) + k $$

In general, the same rules apply when it comes to vertical shifts, horizontal shifts and reflections. The h value is “attached” to the x inside the parentheses and shifts the graph left and right (recall that the signs are opposite the direction the graph moves!!!). The k value is just as before and shifts the graphs up and down. Two graphs below show examples of these.

Two more new things to consider are the **amplitude** and the **period** of the function. The amplitude is the distance from the centerline to a peak or a trough as shown to the left.

***Notice that in this graph, even though the peak reaches a value of 4, the amplitude is only 3, since the distance from the centerline is 3. The graph had a vertical shift of 1 which may change where the graph peaks, but does not change or have any effect on the amplitude.
The period of a function is defined as the distance it moves along the x before it completes one full cycle. One full period cosine for example looks like the graph to the right. The formula for the period is given by

\[ T = \left| \frac{2\pi}{b} \right| \]

For a given function, the period can tell you where your graph starts to repeat. Another way to think about the number that is taking the place of b, is that it tells you how many cycles you have within \(2\pi\) along the x-axis. Why don’t we consider both methods with the equation below.

Let’s look at the equation

\[ f(x) = \sin(2x) \]

The first way we can think about it is that the period, \(T\), can be figured out by using the formula,

\[ T = \left| \frac{2\pi}{b} \right| = \left| \frac{2\pi}{2} \right| = \pi \]

So that means one cycle is completed within a distance of \(\pi\) along the x axis. You can see that in the graph below:

![Graph showing one cycle within a distance of \(\pi\)]

But, thinking about it the second way, we can see that

\[ b = 2 \]

And that means that before the x distance gets to \(2\pi\), there needs to be 2 full cycles of the graph. That is also shown in the graph to the left.

As before, all of these things can happen in one graph and you will need to use all techniques in order to graph these functions correctly.
Example

Graph one period of \(-\cos (3x)\)

Solution

First start with the graph of regular cosine

Then recall that the negative sign flips the graph across the x axis

Then, the 3 inside the parentheses means that in the normal period of cosine (which is 2\(\pi\)) is shrunk by a factor of three. So the new period of the function is \(\frac{2\pi}{3}\). Therefore, the graph looks as follows:
Practice Problems:

Graph each of the following

1. \( y = -\sin(x - \frac{\pi}{4}) \)
2. \( y = \cos(2x + \frac{\pi}{3}) \)
3. \( y = \sin(x) + 1 \)
4. \( y = -\cos(5x) \)
5. \( y = -\sin(x - \frac{\pi}{6}) \)
6. \( y = \sec(x + \pi) \)
7. \( y = -\csc(x + \frac{3\pi}{2}) \)
8. \( y = \tan\left(x - \frac{\pi}{4}\right) + 2 \)
9. \( y = -\cot(2x) - 1 \)
10. \( y = \sin\left(3x - \frac{\pi}{3}\right) + \frac{1}{2} \)
SOLUTIONS
Exponent Simplification and Equations

1. \( x^9 \)
2. \( y^3 \)
3. \( a^{21}b^7 \)
4. \( \frac{1}{p^6q^{18}} \)
5. \( a^4 \)
6. \( 27a^3b^{12} \)
7. \( 1 \)
8. \( -\frac{x^{10}z^{13}}{3y^3} \)
9. \( b^\frac{4}{3} \)
10. \( \frac{4x^7}{yz} \)

Square Root Manipulation

1. \( \frac{\sqrt{7}}{2} \)
2. \( \frac{2\sqrt{3}+9}{3} \)
3. \( 9x^2y + 1 \)
4. \( 4x \)
5. \( \frac{5\sqrt{7}-7}{7} \)
6. \( \sqrt{2} + \frac{1}{2} \)
7. \( 3 - \sqrt{5} \)
8. \( 2 + \sqrt{3} + 3\sqrt{2} + 2\sqrt{6} \)

Imaginary Numbers

1. \( \frac{-6}{13} - \frac{4}{13}i \)
2. \( \frac{-6}{29} + \frac{14}{29}i \)
3. \( 13 \)
4. \( \frac{1}{2} - \frac{3}{2}i \)
5. \( -\frac{1}{5} + \frac{7}{5}i \)
6. a. \( x + y = 2 + 2i \)
   b. \( x - y = -2 + 4i \)
   c. \( x \ast y = 3 + 6i \)
   d. \( \frac{x}{y} = -\frac{3}{5} + \frac{6}{5}i \)
7. a. \( x + y = \frac{7}{10} + 2i\sqrt{3} \)
   b. \( x - y = \frac{3}{10} - 4i\sqrt{3} \)
   c. \( x \ast y = \frac{91}{10} + \frac{13\sqrt{3}}{10}i \)
   d. \( \frac{x}{y} = -\frac{445}{1352} - \frac{85\sqrt{3}}{1352}i \)
8. \( x = \frac{1}{4} + \frac{3\sqrt{7}}{4}i \), \( \frac{1}{4} - \frac{3\sqrt{7}}{4}i \)
9. \( x = i\sqrt{6} , -i\sqrt{6} \)
10. \( t = \frac{1}{5} + \frac{\sqrt{14}}{5}i \), \( \frac{1}{5} - \frac{\sqrt{14}}{5}i \)
Absolute Value Equations

1. 5
2. −5
3. \( x = 4 \text{ and } -8 \)
4. \( x < 2 \text{ or } (-\infty, 2) \)
5. \( x = 0 \)
6. \( x = 3 \text{ and } -3 \)
7. \( x = -1 \text{ and } 3 \)
8. \( x = 2 \text{ and } -2 \)
9. \( x = 15 \text{ and } -15 \)
10. \( x = 5 \text{ and } -5 \)

Logarithms

1. 3
2. 1
3. No Solution
4. 0
5. 0
6. 2
7. \( \log(15) + \log(3) \)
8. \( \log(15) - \log(3) \)
9. \( 10 \times \log(10) \)
10. \( x = \frac{7}{2} \)
11. \( x = \frac{1}{9} \)
12. \( x = 7 e^{24} \)

Higher Level Graphing

1.

2.

3.

4.
Graphing Inequalities

1. $y \leq x$

2. $y < 4x - 1$

3. $y = -2x + 4$

4. $y > \frac{1}{2}x + 3$

5. $x \geq 0$

6. $y = -2.5$

7. $y = \frac{1}{2}x - 1$

8. $y > x^3$
Range and Domain

1. Domain: All Real Numbers,  
   Range: \((0, \infty)\)

2. Domain: \((-\infty, 3) \cup (-3, 3) \cup (3, \infty)\),  
   Range: \((-\infty, +\infty)\)

3. Domain: \([-5, \infty)\),  
   Range: \([0, \infty)\)

4. Domain: \((-\infty, \infty)\),  
   Range: \((0, \infty)\)

5. Domain: \([2, \infty)\),  
   Range: \([3, \infty)\)

6. \(x > 4\)

7. Domain: All Reals,  
   Range: \((0, \infty)\)

8. Domain: All Real Numbers,  
   Range: \((0, \infty)\)

9. Domain: \((-\infty, -2) \cup (-2, \infty)\),  
   Range: \((-\infty, \frac{2}{3}) \cup (\frac{2}{3}, \infty)\)

10. Domain: \((-\infty, \sqrt{5}) \cup (\sqrt{5}, \infty)\),  
    Range: \((-\infty, 1) \cup (1, \infty)\)
SOLUTIONS

Even and Odd Functions

1. Even
2. Even
3. Neither
4. Odd
5. Even
6. Odd
7. Odd
8. Neither
9. Even
10. Neither

Inverses of Functions

1. a. By the definition of inverse functions, the equation $f[g(x)] = x$ is true for each $x$ in the domain of $g$. Therefore, using $x = 13$, we have $f[g(13)] = 13$
   b. Since $\sqrt{5}$ is in the domain of $f$, the definition of inverses functions gives us $g[f(\sqrt{5})] = \sqrt{5}$

2. a. $x = 5$
   b. $x = 7$
3. $x = -1$
4. $f^{-1}(x) = \frac{-5x-1}{3x-1}$
5. $f^{-1}(x) = \frac{x-11}{7}$

6. a. $f(g(x)) = g(f(x))$
   b. $f(g(x)) = g(f(x))$
   c. Assume $x$ is positive,
   \[ g(h(x)) = h(g(x)) \]
   d. $f(g(x)) = g(f(x))$

Composite Functions

1. $x^2 - x - 7$
2. $-x^2 + 5x + 5$
3. $2x^3 - x^2 + 3x + 6$
4. $2x^3 - 7x^2 - 9x + 6$
5. $-x^2 + 2x + 8$
6. $2x^4 - x^3$
7. $\frac{2x-1}{x^3}$
8. $\frac{7-x}{6x}$
9. $\frac{7-t}{6t}$
10. $\frac{1-6y}{7}$
11. $\frac{1-6x}{7}$
12. $f(x) = 2x + 1$ and $g(x) = \frac{3}{\sqrt{x}}$
13. $f(x) = \frac{3}{\sqrt{x}}$ and $g(x) = 2x + 1$
14. $f(x) = \frac{1}{x}$ and $g(x) = 2x + 1$
15. $f(x) = |x|$ and $g(x) = 3x - 1$
SOLUTIONS

Simplification of Fractions Using Quadratic Factoring

1. \(\frac{1}{x+6}\)
2. \(\frac{x+6}{x+5}\)
3. \(\frac{x-12}{x-1}\)
4. \(\frac{1}{x+6}\)
5. \(\frac{1}{3x+1}\)
6. \(\frac{7x+2}{2x+1}\)
7. \(\frac{6x-1}{x-9}\)

Polynomial Long Division

1. \(x + 1\)
2. \(x + 3\)
3. \(5x^2 + 2x + 1\)
4. \(4x^{27} - 10x^4 + 5\)
5. \(x^2 + 3x + 1\) remainder 5 
   \(or\) \(x^2 + 3x + 1 + \frac{5}{x-3}\)
6. \(9x^5 + x^3 + \frac{3}{x^3}\)
7. 5x remainder \(x^3 - 3x + 5\)
   \(or\) 5x + \(\frac{x^3 - 3x + 5}{x^4 + 1}\)

Geometry

1. \(A = 12x^2\)
2. \(A = 4t^2\)
3. \(A = \left(\frac{9}{2} - \frac{\pi}{4}\right)y^2\)
4. \(A = 6Hy\)
5. \(A = \pi x^2\)
6. \(w = 3\)
7. \(A = \frac{1}{2}\)
8. \(x = 4\)
9. \(P = 14z, A = 6z^2\)
10. \(A = 64 - 16\pi\)
Unit Circle and Basic Trigonometry

1. $\frac{-\sqrt{3}}{2}$
2. $-\frac{1}{\sqrt{2}}$
3. $-\frac{2}{\sqrt{3}}$
4. 0
5. Quadrants I and IV
6. 84°
7. $\frac{34\pi}{15}$
8. 240°
9. 1
10. See page 56 to see unit circle

Solving Trigonometric Equations

1. $x = \frac{\pi}{3} ; \frac{5\pi}{3}$
2. $x = \frac{\pi}{6} ; \frac{5\pi}{6}$
3. $x = \frac{\pi}{3} ; \frac{5\pi}{3}$
4. For $\cos\left(\frac{\pi}{4}\right) = \cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$
   the solution is $x = -\frac{\pi}{20}$ and $\frac{\pi}{4}$
5. $x = 0 ; \frac{\pi}{3} ; \frac{5\pi}{3} ; 2\pi$
6. $\frac{\pi}{6} ; \frac{5\pi}{6} ; \frac{3\pi}{2}$
7. $t = \sin^{-1}\left(\frac{5-\sqrt{13}}{-6}\right)$ and
   $t = \sin^{-1}\left(\frac{5+\sqrt{13}}{-6}\right)$
8. $x = \frac{\pi}{6} ; \frac{5\pi}{6} ; \frac{7\pi}{6} ; \frac{11\pi}{6}$
9. $x = \frac{\pi+2}{6} ; \frac{3\pi+2}{6}$
10. $x = \pm \frac{\pi}{12}$

Geometric Trigonometry

1. $\sin \theta = \frac{4}{5}, \sec \theta = \frac{5}{3}, \tan \theta = \frac{4}{3}$
2. $\sin \theta = \frac{6}{\sqrt{37}}, \sec \theta = \sqrt{37}$, $\tan \theta = 6$
3. $\theta = 45°$
4. $\theta = 30°$
5. $\theta = 60°$
6. $x = -5 + 10\sqrt{3}$ or ~12.32
7. $x = \frac{14}{\sqrt{3}}$
8. $x = 6$
9. $x = \pm \sqrt{13}$
10. $x = 90°$

Trigonometric Graphing

1. $y = -\sin(x - \frac{\pi}{2})$
2. $y = \cos(2x + \frac{\pi}{2})$
The purpose of this book is to prepare students wishing to take the college level math test in order to bypass algebra or trigonometry. Those wishing to test straight into calculus should know all material in this book well. The material has been put together by MSU Denver Student Academic Success Tutoring Center.

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